

METHOD OF ANALYSIS AND ASSESSMENT OF READINESS OF TECHNICAL OBJECTS USING MARKOV THEORY

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General Remarks –introduction

1. Assumption for the mathematical model
2. Model for calculating helicopter readiness for the Markov chain
3. Model for calculating helicopter readiness for the Markov process
4. Limit probabilities for discrete and continuous time for Mi-8
5. Summary and Conclusions

Introductory remarks on the operation of military facilities

- 1) Small numbers of military objects in systems (limit the reliability of estimates and analysis)
- 2) Complicated procedure instructions and regulations, as well as mainly paper operation records, complicate and make it impossible to determine the phase space of the process
- 3) Budget restrictions that cause renewal downtime and minimize planned operation time and the number of transitions between states

GENERAL REMARKS

The measures of the readiness constitute the numerical characteristics of the processes performed in a given operating system and can be expressed as:

- 1) momentary probabilities of being in the particular states of the readiness
- 2) stationary (asymptotic, ergodic) probabilities of being in the particular states in a definite period of time
- 3) average values and variances in the residence times in the desired states within a definite period of time
- 4) statistical frequency distributions or probability distributions of the interstate transition times
- 5) frequency or intensity of the transitions between the different operational states

GENERAL REMARKS

The determination of the readiness measures of the technical object usually involves two stages

STAGE 1

the process
identification and a
development of the
calculation
algorithms

STAGE 2

the measurements,
collection, processing and
control of the databases
enabled to be imported
through the analytical
software in order to
determine the values of
the readiness parameters
used in the models

1. ASSUMPTIONS FOR THE MATHEMATICAL MODEL

1. A technical object may at any time be in only one of the possible operation states, the set of which creates a countable and finite process phase space
2. The phase space of an object is a finite, discrete set of disjoint operating states. The maximum number of states is determined by the numerical limitations of commercial analysis software
3. Returns ($S_i \rightarrow S_i$ transitions) are prohibited during the operation process
4. Deterministic, environmental and external factors influencing the operation process are known
5. Times of changes of object states are recorded with any accuracy
6. The process described is a process without memory brak pamięci oznacza

1. ASSUMPTIONS FOR THE MATHEMATICAL MODEL

Table 1. Part from the raw (unprocessed) helicopter database

No.	DATE	TIME	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈	S ₉	K
1	Jan2.19	08:00:00	1									1
2	Jan2.19	14:00:00									1	2
3	Jan10.19	06:00:00						1				3
4	Jan15.19	06:10:00	1									4

Table 2. Part from helicopter source database

No.	DATE	TIME	S ₁	S ₂	S ₃	S ₄	S ₅	S ₆	S ₇	S ₈	S ₉	K
1	43467	0.333333	1									1
2	43467	0.583333									1	2
3	43475	0.25						1				3
4	43480	0.256944	1									4

where: No. - number of an entry in the object register, DATE - date of state entry (Table 1) in Excel format (Table 2),
 TIME - time of state entry (Table 1) in Excel format (Table 2),
 K - the number of entry into a state
 S₁;...;S₉ - auxiliary binary variables with value 1 only for the state with an entry number No.

1. ASSUMPTIONS FOR THE MATHEMATICAL MODEL

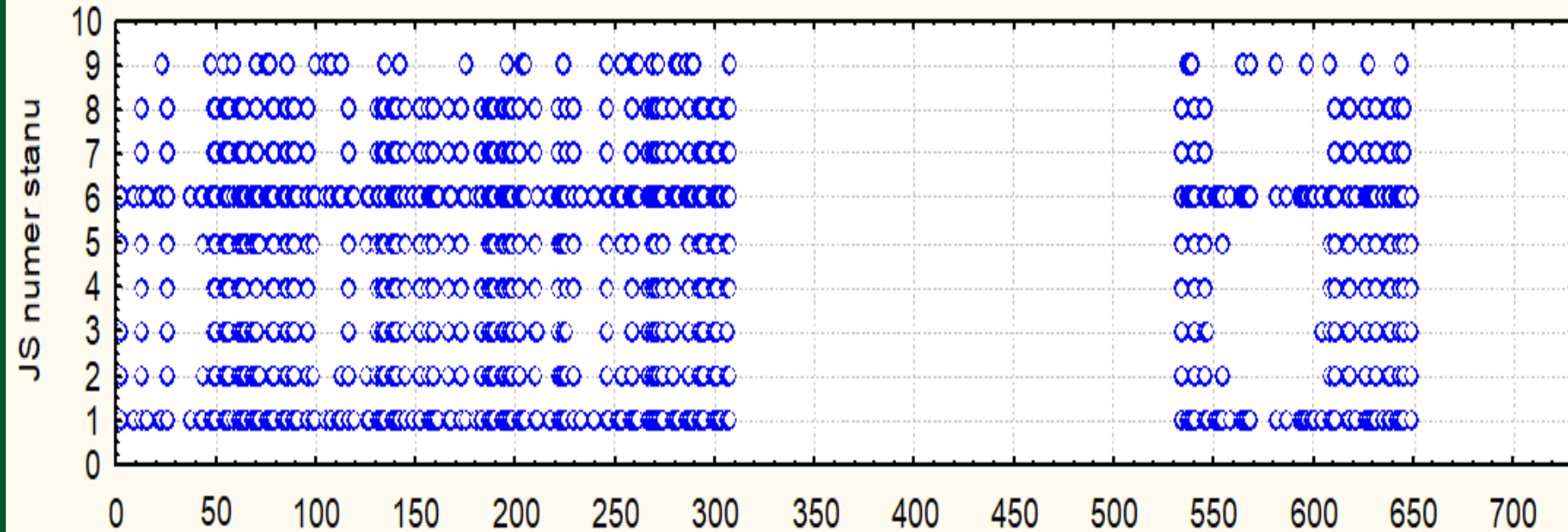


Fig 1. Phase trajectory in physical time of the Mi-8 helicopter

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

$S_i \rightarrow S_j$	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9
S_1	0	1	0	0	0	1	1	0	1
S_2	1	0	1	1	1	0	1	0	0
S_3	1	0	0	1	1	0	1	0	0
S_4	0	1	1	0	0	0	1	0	0
S_5	1	0	1	0	0	1	0	0	0
S_6	1	0	1	0	0	0	0	0	0
S_7	0	1	1	1	1	0	0	1	0
S_8	0	0	0	0	0	0	1	0	0
S_9	1	1	0	0	0	1	0	0	0

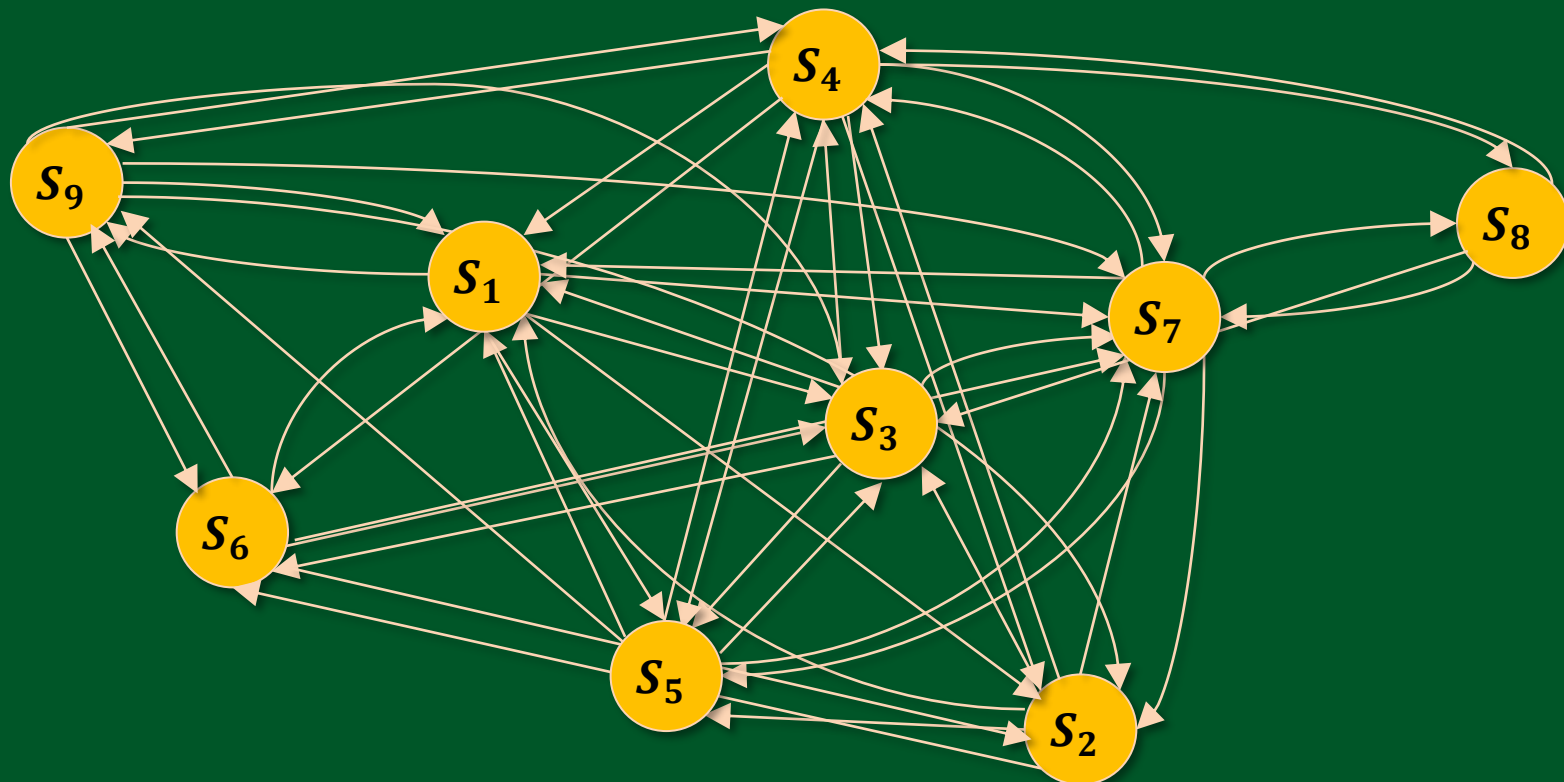
Table 3. Allowed transitions matrix for helicopter exploitation process

where:

0 – means forbidden transitions

1 – means transitions allowed

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN



S_1 - diagnostics; S_2 - test execution; S_3 - supply; S_4 - readiness with pilot; S_5 - readiness without pilot;
 S_6 - being on duty in a hangar; S_7 - working on the ground; S_8 - task execution; S_9 - unsuitability.

Fig. 2. Graph of allowed transitions for the nine-state operational model for the whole helicopter test

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

S_1 – diagnostics

S_2 – test execution

S_3 – supply (refueling)

S_4 – readiness with pilot

S_5 – readiness without pilot

S_6 – being on duty in a hangar

S_7 – working on the ground

S_8 – task execution

S_9 – unsuitability (repairs and inspections at the air base)

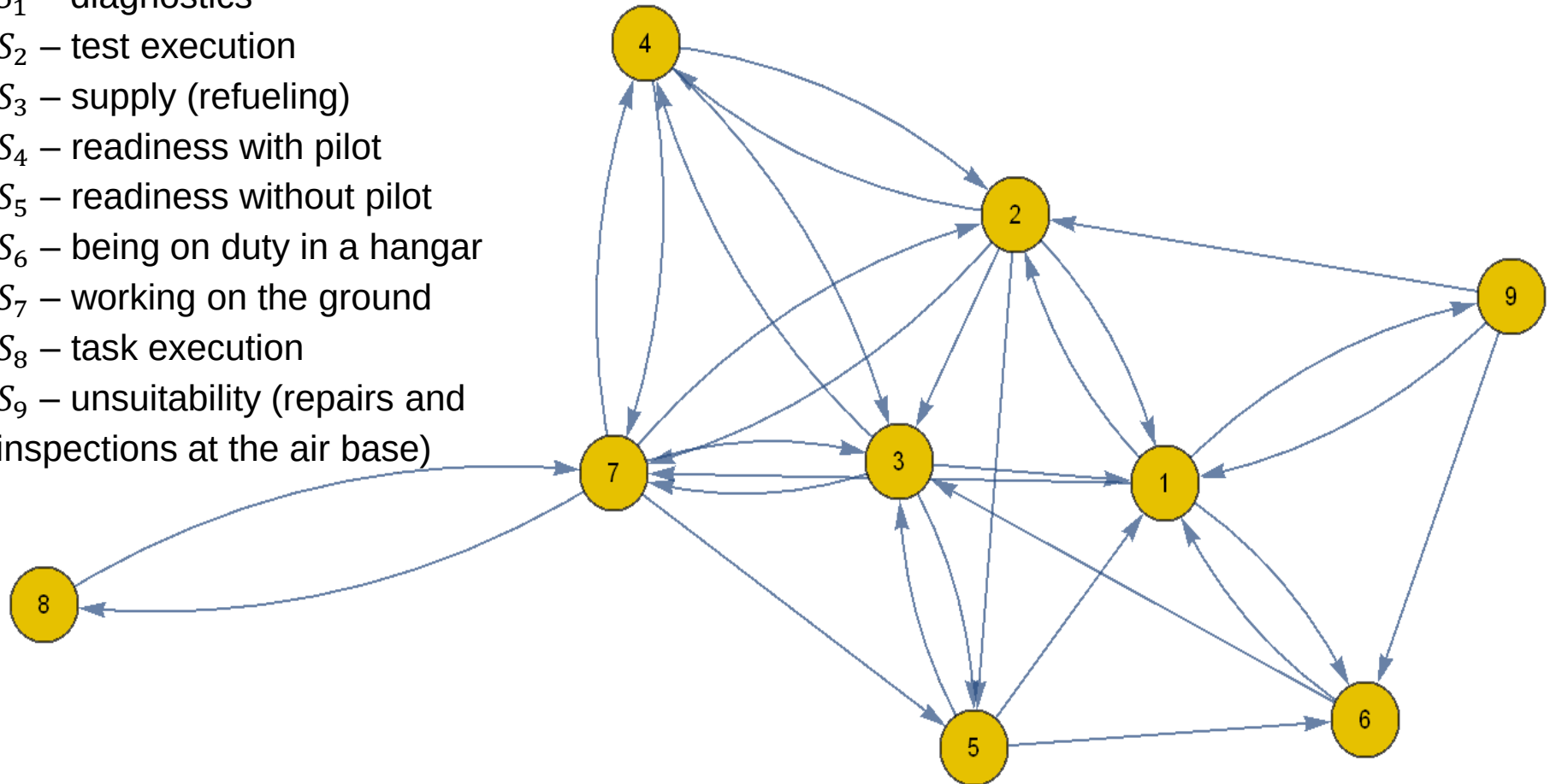


Fig. 3. Graph of allowed transitions for the nine-state operation model of a single Mi-8 helicopter

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

Table 4. The empirical numbers n_{ij} of the interstate transitions between the $S_i \rightarrow S_j$ states for Mi-8 helicopter

$p_i \rightarrow p_j$	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
p_1	0	72	0	0	0	70	3	0	16
p_2	2	0	17	119	25	0	2	0	0
p_3	35	0	0	24	39	0	1	0	0
p_4	0	90	1	0	0	0	76	0	0
p_5	4	0	25	0	0	41	0	0	0
p_6	126	0	6	0	0	0	0	0	0
p_7	0	2	50	24	6	0	0	95	0
p_8	0	0	0	0	0	0	95	0	0
p_9	1	1	0	0	0	15	0	0	0

Source: own development.

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

The first stage in the construction of Markov process with discrete time is the estimation of the probabilities of transitions, as the $\hat{p}_{ij}$ estimator values of p_{ij} elements and the P probabilities matrix. The values of these estimators from the test sample are the frequencies w_{ij} of transitions from the state S_i to the state S_j , calculated according to Eqn (1):

$$\hat{p}_{ij} = w_{ij} = n_{ij} / n_i \quad (1)$$

where:

w_{ij} - frequency of transitions from the state S_i to the state S_j ,

n_{ij} - number of transitions from the state S_i to the state S_j ,

n_i - number of all transitions (exits) from the state S_i .

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

For the analysed process of operation, a 9x9 matrix of inter-state transitions P is created:

$$P[p_{ij}] = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & p_{16} & 0 & 0 & p_{19} \\ 0 & 0 & p_{23} & p_{24} & p_{25} & 0 & p_{27} & 0 & 0 \\ p_{31} & 0 & 0 & p_{34} & p_{35} & 0 & 0 & 0 & 0 \\ 0 & p_{42} & p_{43} & 0 & 0 & 0 & p_{47} & 0 & p_{59} \\ p_{51} & 0 & p_{53} & 0 & 0 & p_{56} & 0 & 0 & 0 \\ p_{61} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{71} & p_{72} & p_{73} & p_{74} & p_{75} & 0 & 0 & p_{78} & 0 \\ 0 & 0 & p_{83} & 0 & 0 & 0 & p_{87} & 0 & 0 \\ p_{91} & p_{92} & p_{93} & 0 & 0 & p_{96} & 0 & 0 & 0 \end{bmatrix} \quad (2)$$

Table 5 summarises the empirical probabilities of helicopter interstate transitions during the two-year study period.

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

Table 5. Empirical frequencies (probabilities) p_{ij} of interstate transitions for Mi-8 helicopter

$p_i \rightarrow p_j$	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
p_1	0	0,447204	0	0	0	0,434782	0,018633	0	0,099378
p_2	0,012121	0	0,103030	0,721212	0,151515	0	0,012121	0	0
p_3	0,353535	0	0	0,242424	0,393939	0	0,010101	0	0
p_4	0	0,538922	0,005988	0	0	0	0,455089	0	0
p_5	0,057142	0	0,357142	0	0	0,585714	0	0	0
p_6	0,954545	0	0,045454	0	0	0	0	0	0
p_7	0	0,011299	0,282485	0,135593	0,033898	0	0	0,536723	0
p_8	0	0	0	0	0	0	1	0	0
p_9	0,058823	0,058823	0	0	0	0,882352	0	0	0

$$\hat{p}_{ij} = w_{ij} = n_{ij} / n_i = 72/161 = 0,4472$$

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN

$$\lim_{n \rightarrow \infty} p_{ij}(n) = \sum_i p_i p_{ij} = p_j \Leftrightarrow P^T [p_j] = [p_j], \quad \sum_j p_j = 1 \quad (3)$$

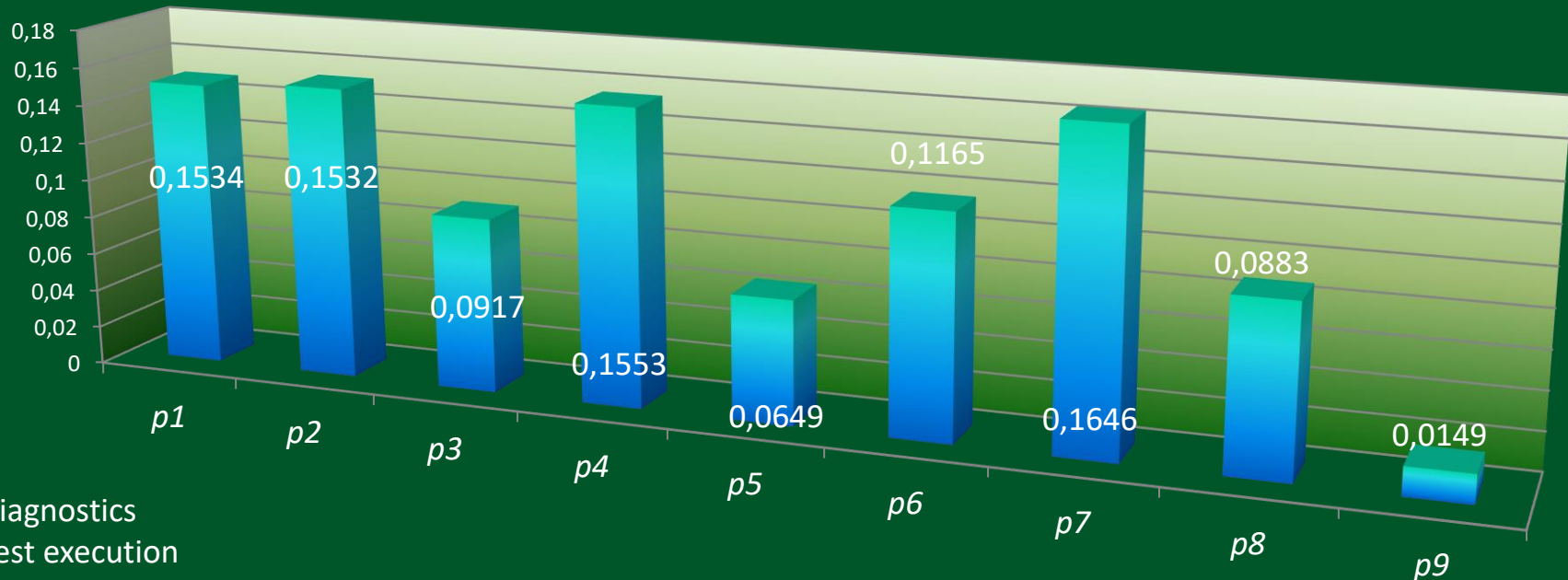
where:

P^T - transposed interstate transition P matrix $P = [p_{ij}; i, j \in S]$; $[p_j]$ - vector of ergodic probabilities;
 p_{ij} - probability of transition from i to j state

$$\left\{ \begin{array}{l} 0,012121p_2 + 0,353535p_3 + 0,057142p_5 + 0,954545p_6 + 0,058823p_9 - p_1 = 0 \\ 0,447204p_1 + 0,538922p_4 + 0,011299p_7 - 0,058823p_9 - p_2 = 0 \\ 0,103030p_2 + 0,005988p_4 + 0,357142p_5 + 0,045454p_6 + 0,282485p_7 - p_3 = 0 \\ 0,721212p_2 + 0,242424p_3 + 0,135593p_7 - p_4 = 0 \\ 0,151515p_2 + 0,393939p_3 + 0,033898p_7 - p_5 = 0 \\ 0,434782p_1 + 0,585714p_5 + 0,882352p_9 - p_6 = 0 \\ 0,018633p_1 + 0,012121p_2 + 0,010101p_3 + 0,455089p_4 + 1p_8 - p_7 = 0 \\ 0,536723p_7 - p_8 = 0 \\ 0,099378p_1 - p_9 = 0 \end{array} \right. \quad (4)$$

$$\text{przy } \sum_{i=1}^9 p_j = 1 \quad (5) \quad 16$$

2. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV CHAIN



S_1 – diagnostics
 S_2 – test execution
 S_3 – supply (refuelling)
 S_4 – readiness with pilot
 S_5 – readiness without pilot
 S_6 – being on duty in a hangar
 S_7 – working on the ground
 S_8 – task execution
 S_9 – unsuitability (repairs and inspections at the air base)

Fig. 4. Boundary probabilities $p_j(n)$ of the Markov chain for the helicopter

3. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV PROCESS

Table 6. Matrix of intensities Λ for helicopter transitions

$\lambda_i \rightarrow \lambda_j$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
λ_1	-0,096710	0,010142	0	0	0	0,003961	0,016216	0	0,06639
λ_2	0,0799	-0,387914	0,058620	0,065348	0,050607	0	0,133333	0	0
λ_3	0,313948	0	-0,995498	0,352768	0,288782	0	0,04	0	0
λ_4	0	0,047199	0,013889	-0,078749	0	0	0,017661	0	0
λ_5	0,008681	0	0,012773	0	-0,026391	0,004936	0	0	0
λ_6	0,000319	0	0,000641	0	0	-0,000961	0	0	0
λ_7	0	0,086956	0,163934	0,172661	0,176471	0	-0,761588	0,161564	0
λ_8	0	0	0	0	0	0	0,015936	-0,015936	0
λ_9	0,002816	0,0025	0	0	0	0,00344	0	0	-0,008757

3. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV PROCESS

For homogeos Markov procesess $X(t)$ non-diagonal intensities are calculated according to :

$$\lambda_{ij} = \frac{1}{\bar{t}_{ij}} \quad \bar{t}_{ij} = \sum_{k=1}^n n \bar{t}_{ij} / N \quad (4)$$

where: $i, j \in 1, \dots, 9$; $\bar{t}_{ij}$ means the average time spent in the state S_i before the state S_j

The diagonal intensities $\lambda_{ii} \leq 0$ for $i = j$ are defined as the sum cofactor of the intensities of transitions from S_i state and calculated as:

$$\lambda_{ii} + \sum_j \lambda_{ij} = 0 \quad \text{from here:} \quad \lambda_{ii} = - \sum_j \lambda_{ij} \quad (5)$$

3. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV PROCESS

The systems of CH-K-S equations have the following matrix form:

$$\frac{d}{dt}P(t) = \Lambda \cdot P(t) \wedge \left(\sum_j p_j = 1\right) \quad (6)$$

where: $P(t)$ - a column vector of the probabilities of the process being in particular states;
 Λ – transition intensity matrix for the 9-state model of the process; $\sum_j p_j = 1$ condition of normalization of the system.

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \\ p_9 \end{bmatrix} \cdot \begin{bmatrix} -\lambda_{11} & \lambda_{12} & 0 & 0 & 0 & \lambda_{16} & \lambda_{17} & 0 & \lambda_{19} \\ \lambda_{21} & -\lambda_{22} & \lambda_{23} & \lambda_{24} & 0 & 0 & 0 & 0 & 0 \\ \lambda_{31} & 0 & -\lambda_{33} & \lambda_{34} & \lambda_{35} & 0 & \lambda_{37} & 0 & 0 \\ 0 & \lambda_{42} & \lambda_{43} & -\lambda_{44} & 0 & \lambda_{46} & \lambda_{47} & \lambda_{48} & \lambda_{49} \\ \lambda_{51} & 0 & \lambda_{53} & 0 & -\lambda_{55} & \lambda_{56} & 0 & 0 & 0 \\ \lambda_{61} & 0 & \lambda_{63} & 0 & 0 & -\lambda_{66} & 0 & 0 & 0 \\ \lambda_{71} & 0 & \lambda_{73} & \lambda_{74} & \lambda_{75} & 0 & -\lambda_{77} & \lambda_{78} & 0 \\ 0 & 0 & \lambda_{83} & \lambda_{84} & 0 & 0 & \lambda_{87} & -\lambda_{88} & 0 \\ \lambda_{91} & \lambda_{92} & 0 & 0 & 0 & \lambda_{96} & 0 & 0 & -\lambda_{99} \end{bmatrix} = \begin{bmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \\ \dot{p}_3(t) \\ \dot{p}_4(t) \\ \dot{p}_5(t) \\ \dot{p}_6(t) \\ \dot{p}_7(t) \\ \dot{p}_8(t) \\ \dot{p}_9(t) \end{bmatrix}$$

It is a homogeneous system with an infinite number of solutions, among which there may be solutions that meet the condition of standardization:

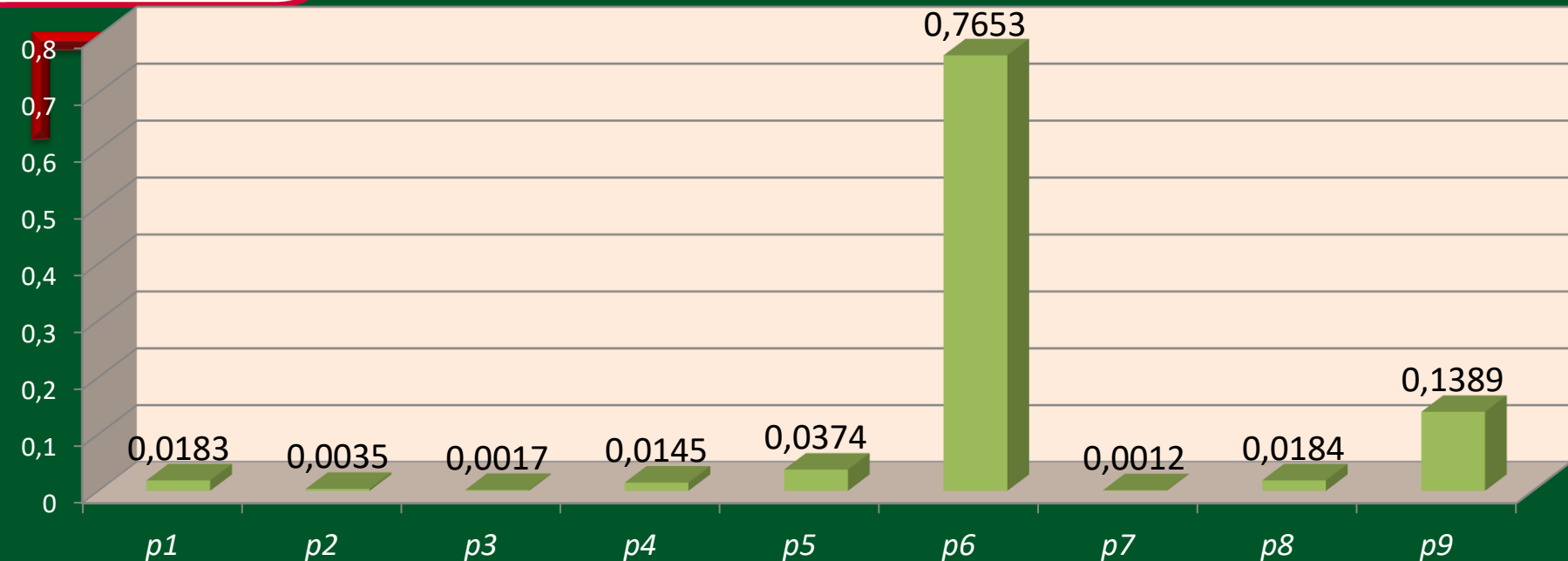
$$\sum_{j=1}^9 \dot{p}_j(t) = 1$$

3. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV PROCESS

$$\begin{cases} -0,096710p_1 + 0,0799p_2 + 0,3139p_3 + 0,008681p_5 + 0,000319p_6 + 0,002816p_9 = p_1'(t) \\ 0,010142p_1 - 0,387914p_2 + 0,047199p_4 + 0,086956p_7 + 0,0025p_9 = p_2'(t) \\ 0,058620p_2 - 0,995498p_3 + 0,013889p_4 + 0,012773p_5 + 0,000641p_6 + 0,163934p_7 = p_3'(t) \\ 0,065348p_2 + 0,352768p_3 - 0,078749p_4 + 0,172661p_7 = p_4'(t) \\ 0,050607p_2 + 0,288782p_3 - 0,026391p_5 + 0,176471p_7 = p_5'(t) \\ 0,003961p_1 + 0,004936p_5 - 0,000961p_6 + 0,00344p_9 = p_6'(t) \\ 0,016216p_1 + 0,133333p_2 + 0,04p_3 + 0,017661p_4 - 0,761588p_7 + 0,015936p_8 = p_7'(t) \\ 0,161564p_7 - 0,015936p_8 = p_8'(t) \\ 0,06639p_1 - 0,008757p_9 = p_9'(t) \end{cases} \quad (8)$$

$$\text{and } \sum_{j=1}^9 \dot{p}_j(t) = 1$$

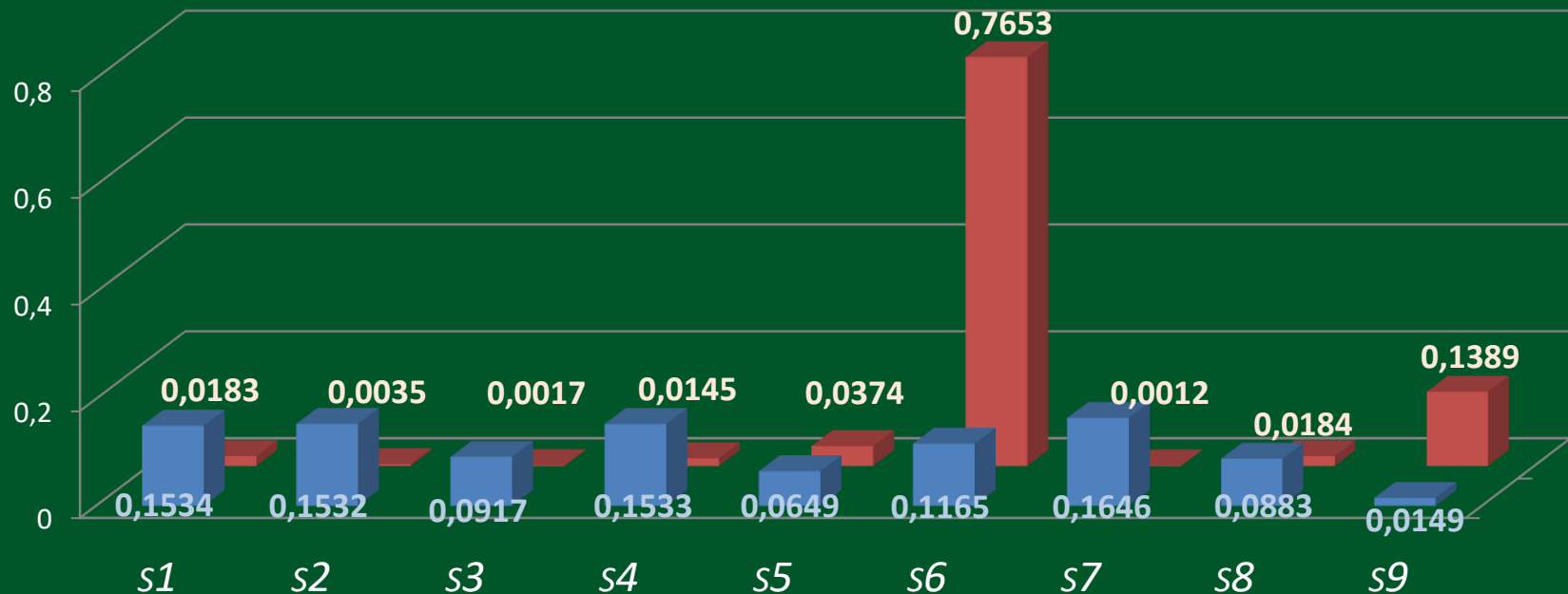
3. MODEL FOR CALCULATING HELICOPTER READINESS FOR THE MARKOV PROCESS



- S_1 – diagnostics
- S_2 – test execution
- S_3 – supply (refuelling)
- S_4 – readiness with pilot
- S_5 – readiness without pilot
- S_6 – being on duty in a hangar
- S_7 – working on the ground
- S_8 – task execution
- S_9 – unsuitability (repairs and inspections at the air base)

Fig. 5. Boundary probabilities $p_j(t)$ of the Markov process for the helicopter

4. BORDER PROBABILITIES FOR DISCRETE AND CONTINUOUS TIME FOR Mi-8



- S_1 – diagnostics
- S_2 – test execution
- S_3 – supply (refuelling)
- S_4 – readiness with pilot
- S_5 – readiness without pilot
- S_6 – being on duty in a hangar
- S_7 – working on the ground
- S_8 – task execution
- S_9 – unsuitability (repairs and inspections at the air base)

■ $p_j(n)$
■ $p_j(t)$

Fig.6. Ergodic probabilities of Mi-8 helicopter in discrete $p_j(n)$ and continuous $p_j(t)$ time

5. SUMMARY AND CONCLUSIONS

Analyzing the results regarding the limiting probabilities of Mi-8 helicopters in relation to the Markov process in discrete time $p_j(n)$ and continuous time $p_j(t)$ the following final conclusions can be formulated:

a) FOR DISCRETE TIME:

- the highest entry probabilities were observed for the state: S_7 (working on ground)
- almost identical for states: S_1 (diagnostics), S_2 (test execution) and S_4 (readiness with pilot) due to the fact that these processes are positively correlated,
- slightly lower entry probabilities apply to the following states: S_6 (being on duty in a hangar), S_3 (supply-refuelling) oraz S_8 (task execution)
- the lowest entry probabilities are for states: S_9 (unsuitability-repairs and inspections at the air base) and S_5 (readiness without pilot)

b) FOR CONTINUOUS TIME:

- calculated functional readiness index for Mi-8 helicopters understood as $\sum_{j=4}^8 p_j$ is 0,837. Therefore, it is high, which proves that the operation process is properly carried out from the point of view of supervision of technical facilities equipped with Blot. The highest residence probabilities were observed for states S_6 (being on duty in a hangar) and S_9 (unsuitability). The probabilities of staying in other states are short-term and do not have a significant impact on the calculated functional readiness index.

THANK YOU FOR
YOUR ATTENTION!